

Appendix A Decibels

Communication systems often consist of many different blocks, connected together in a chain so that a signal must travel through one after another. Fig. A-1 shows the *block diagram* of such a chain, consisting of an amplifier, a cable, an attenuator, another amplifier, and a filter. An input signal must travel through all five of these before it arrives at the output.

Typically, each of these blocks will affect the signal in some way. For example, an amplifier will increase its amplitude, whereas an attenuator will decrease it. The cable will also probably decrease it (though perhaps not as much as an attenuator), and so will the filter.

For each of these blocks, we can therefore define a gain or loss. The gain would be defined as

$$\text{gain} = \frac{\text{output magnitude}}{\text{input magnitude}}$$

Another form of this same equation is

$$\text{output magnitude} = \text{gain} \times \text{input magnitude}$$

The magnitudes of the input and output could be measured in units such as volts, amperes, or watts, and these could be rms, peak, peak-to-peak, average, or even possibly something else. In each case, however, the units in the numerator and denominator must be the same, and they determine whether the gain is called a *voltage gain*, *power gain*, or *current gain*. Since the gain is found by calculating a ratio, it is also often called a *gain ratio*.

For example, if an amplifier provides 5 volts of output when the input is 1 volt, then the voltage gain is 5/1 or 5. Similarly, if 1 watt of input power produces 10 watts of output power, the power gain is 10/1 or 10. (But note: if an input of 10 milliwatts produces an output of 100 watts, then the gain is not 10, but 10,000. You cannot simply divide the 100-watt output by the 10-mw input, because the units are not the same!)

In an amplifier, the output is usually (but not always!) larger than the input, and so the gain is generally greater than 1. A cable's or attenuator's output, on the other hand, is usually smaller than the input. For exam-

ple, the input might be 10 watts while the output might be just 5 watts. The gain would then be

$$\text{Power gain} = \frac{5 \text{ watts}}{10 \text{ watts}} = 0.5$$

Amplifier gains are thus generally larger than 1, while attenuator gains are smaller than 1. Although the attenuator exhibits a loss, we can still refer to it as a gain as long as the gain ratio is smaller than 1.

The gains in our examples have so far been simple numbers such as 5, or 10, or 0.5. In a real communications system, however, the gains might be numbers such as 0.00037 or 300,000. This can lead to quite large differences between input and output magnitudes.

Let us now return to Fig. A-1, and suppose that amplifier #1 has a gain of 100; the cable has a gain of 0.15; amplifier #2 has a gain of 13; the attenuator has a gain of 0.00001, and the filter has a gain of 0.707. The gain of the entire chain of five blocks is the *product of all these separate gains*:

$$\text{gain} = 100 \times 0.15 \times 13 \times 0.00001 \times 0.707 = 0.00137865$$

We can show that this is true by assuming some specific input, such as 1 watt. The first amplifier, with a gain of 100, multiplies this input to 100 watts; the cable's gain of 0.15 (which is really a loss) reduces this to 15 watts; the second amplifier's gain of 13 multiplies this to 195 watts, and so on. We thus eventually get 0.00137865 watts. (There is a problem with this reasoning — if we're not careful, we can get numbers which are larger or smaller than the actual physical device is able to provide. For example, an actual first amplifier might not be able to output 100 watts. Still, the calculation is useful just to show us how the overall gain of a system is calculated.)

In the real world of communications, this entire approach often becomes awkward because some of these gains are extremely large or extremely small numbers; the multiplications then often involve many digits and decimal places. The *decibel* (abbreviated dB) solves this problem in two ways — by converting the large (or small) gains into manageable numbers, and by using logarithms to change multiplication into addition and subtraction.

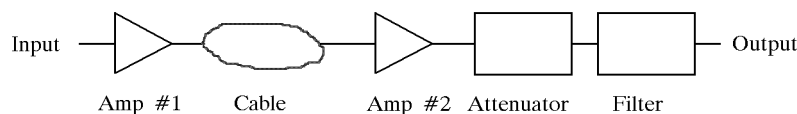


Fig. A-1. A typical communication system

Definition of dB

Since decibels originated in telephone and wire communications, the decibel is named in honor of Alexander Graham Bell, the inventor of the telephone. The unit is actually the *bel*, but

the *decibel* (the prefix *deci* means a tenth) is a more useful unit for most measurements, and no one actually uses the bel.

The decibel is not an absolute unit for measuring anything; rather, it is a way of comparing two measurements and showing how one differs from the other.

The formula to convert a gain ratio into decibels is slightly different for power and voltage or current. We will discuss the voltage and current cases later; in the meantime, the *power* gain in decibels is defined as

$$\text{gain}_{\text{dB}} = 10 \log_{10} \left(\frac{\text{Output power}}{\text{Input power}} \right) = 10 \log_{10} (\text{gain ratio})$$

For example, if amplifier #1 in Fig. A-1 has a power gain of 100, then its power gain in dB is

$$\text{gain}_{\text{dB}} = 10 \log_{10} (100) = 10 \times 2 = 20 \text{ dB}$$

The attenuator with a power gain ratio of 0.00001, on the other hand, becomes

$$\text{gain}_{\text{dB}} = 10 \log_{10} (0.00001) = 10 \times (-5) = -50 \text{ dB}$$

At this point, it might be useful to remember the definition of the “ $\log_{10} ()$ ” notation, which is pronounced as “log, base 10, of”. The logarithm, base 10, of the number inside the parentheses is the power to which we must raise the base 10 to equal the number. For example, 100 is 10^2 , so we must raise 10 to the power 2 to equal 100. Hence the logarithm (base 10) of 100 is 2. Likewise, 0.00001 is 10^{-5} , so \log_{10} of 0.00001 is -5 . Logs of numbers that are powers of 10 are fairly easy to find; logs of other numbers are tougher, and usually require a calculator. For example, amplifier #2 in Fig. A-1 has a gain ratio of 13; this translates to

$$\text{gain}_{\text{dB}} = 10 \log_{10} (13) = 10 \times 1.114 = 11.14 \text{ dB}$$

The dB gains for the other two blocks in Fig. A-1 are found the same way. For the cable whose gain ratio is 0.15, the dB gain is

$$\text{gain}_{\text{dB}} = 10 \log_{10} (0.15) = 10 \times (-0.824) = -8.24 \text{ dB}$$

and the filter with a gain ratio of 0.707 has a dB gain of

$$\text{gain}_{\text{dB}} = 10 \log_{10} (0.707) = 10 \times (-0.15) = -1.5 \text{ dB}$$

Note four things:

- The logarithm of a number greater than 1 is positive. Hence positive decibels represent a real gain, such as an amplifier.
- The logarithm of a number smaller than one is negative. Hence negative decibels represent a loss or attenuation.

- The logarithm of 1 is 0; that is, you raise 10 to the zero power to make it equal to 1. A device that has neither gain nor loss (i.e., its output is the same as its input) therefore has 0 dB gain.
- Only positive numbers have logarithms; logarithms of negative numbers do not exist. If the gain ratio of a block is negative (which means that the output has a different polarity from the input — for example, if 1 volt input gives -100 volts output, then the gain ratio would be -100) then you must ignore the minus sign in calculating the dB gain or loss. Thus gain ratios of $+100$ and -100 both give a 20 dB gain.

We can find the total gain of all five blocks in Fig. A-1 in two ways. If we use the gain ratio of 0.00137865 that we calculated previously, the gain would be

$$\text{gain}_{\text{dB}} = 10 \log_{10} (0.00137865) = 10 \times (-2.86) = -28.6 \text{ dB}$$

The other way is to add up the individual dB gains that we calculated earlier:

Amplifier #1	+20 dB
Cable	-8.24 dB
Amplifier #2	+11.14 dB
Attenuator	-50 dB
Filter	-1.5 dB
Total gain	-28.6 dB

This answer is the same as we calculated earlier.

In most cases, converting to and from decibels requires a calculator. Sometimes, though, it is possible to do approximate conversions with the use of a table, such as this:

Gain ratio	Power gain in dB
1,000,000	+60
100,000	+50
10,000	+40
1000	+30
100	+20
10	+10
2	+3
1	0
.5	-3
.1	-10
.01	-20
.001	-30
.0001	-40
.00001	-50
.000001	-60

We can use the table to calculate the approximate dB gain for a variety of devices. For example, suppose an amplifier has a gain of 4; what is its gain in dB? Now that we know that gain ratios multiply to give total gain, we can see that the two circuits in Fig. A-2 provide equal gains: at the top we have a single amplifier with a gain of 4, while at the bottom we have two amplifiers, each with a gain of 2, whose total gain is therefore $2 \times 2 = 4$. Referring to the above table, a gain of 2 is equivalent to +3 dB. With two such amplifiers in cascade (which means "one after the other"), the gain in dB is therefore 3 dB + 3 dB = 6 dB.

For a second example: a power gain of 50 is the same as having an amplifier with a gain of 100, followed by an attenuator with a gain of 0.5, since the total gain is then $100 \times 0.5 = 50$. In the table, a gain of 100 is +20 dB while the gain of 0.5 is -3 dB, so the total gain is 17 dB.

Even some fairly complicated gains are easy to figure if you can guess how to make them up out of their components. For example, suppose an amplifier has a power gain of 11 dB; what is that in terms of a gain ratio? If you're lucky enough to notice that

$$11 = 20 - 3 - 3 - 3$$

then you see that an amplifier with a power gain ratio of 100 (20 dB), followed by three attenuators with a power gain of 0.5 (-3 dB each) gives the same result. The gain ratio is therefore

$$100 \times 0.5 \times 0.5 \times 0.5 = 12.5$$

You may find this easier to solve if you substitute $\frac{1}{2}$ for the 0.5, like this:

$$100 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = 12.5$$

since one-half of 100 is 50, one-half of 50 is 25, and one-half of 25 is 12.5.

Converting dB to a gain ratio

Another way to convert dB back to a ratio is to solve the dB equation backward. For a gain of 11 dB, for exam-

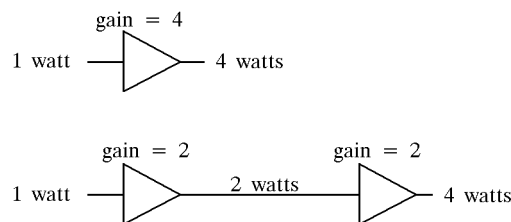


Fig. A-2. Two different ways to get a gain of 4

ple, temporarily let X stand for the gain ratio. The equation is then

$$11 \text{ dB} = 10 \log_{10} X$$

$$1.1 = \log_{10} X$$

From the definition of a logarithm, 1.1 is the power to which we must raise 10 to equal X . In other words,

$$10^{1.1} = X$$

to which a calculator gives the answer as 12.59 (with, again, a small and usually not significant difference from the 12.5 we got earlier.)

A More general use for dB

Rather than defining power gain in dB as

$$\text{Power gain}_{\text{dB}} = 10 \log_{10} \left(\frac{\text{Output power}}{\text{Input power}} \right)$$

we could use the words "before" and "after", as in

$$\text{Power gain}_{\text{dB}} = 10 \log_{10} \left(\frac{\text{Power after}}{\text{Power before}} \right)$$

If you interpret the words "before" and "after" in terms of position, then "before" can mean the input signal before it enters an amplifier or other device, while "after" can mean the output signal after it comes out. In that case we return to the original definition of output power divided by the input power.

But "before" and "after" can also be interpreted in terms of time. For example, "before" could be a measurement at some location before we made some adjustments or repair, while "after" can be a measurement *at the same location* but after the adjustment or repair has been made. In this sense, a typical use of decibels might be to say "Increase the frequency setting until the output signal decreases by 3 dB." Since we've already seen in the previous table that -3 dB is a power gain ratio of 0.5 (and "decreases by 3 dB" means a change of *minus* 3 dB!), this sentence is the same as saying "Increase the frequency until the output power drops to one-half of its previous value."

In some cases, "before" and "after" can be interpreted in terms of both position and time. For example, "before" could mean "last week, over there", while "after" could mean "here, right now". This interpretation is particularly useful for defining the dBm.

dBm

Imagine that sometime in the past you had a device which output 1 milliwatt of power; this becomes the

“before” reference against which we shall compare other powers.

Suppose that we now have before us an amplifier whose output is 100 milliwatts. Since 100 mw is 100 times stronger than 1 mw, and since a power ratio of 100 is +20 dB, we could therefore say that our 100 milliwatt power is “20 dB more than a milliwatt”, or that it is “+20 db compared with one milliwatt.” After a while we get tired of constantly saying “dB compared with a milliwatt” and so we abbreviate the phrase to dBm.

The dBm is therefore simply a way of specifying how a given power compares with one milliwatt. We can thus write the equation for dBm as

$$P_{dBm} = 10 \log_{10} \left(\frac{P}{1 \text{ mw}} \right)$$

For example, 1 watt is equivalent to 30 dBm since 1 watt is 1000 milliwatts (which is 30 db more than *one* milliwatt), and so

$$P_{dBm} = 10 \log_{10} \left(\frac{1000 \text{ mw}}{1 \text{ mw}} \right) = 10 \times 3 = 30 \text{ dBm}$$

Negative dBm values indicate that the power level is less than 1 mW. For example, 10 microwatts is 0.01 milliwatt, which translates to -20 dBm as follows

$$P_{dBm} = 10 \log_{10} \left(\frac{.01 \text{ mw}}{1 \text{ mw}} \right) = 10 \times (-2) = -20 \text{ dBm}$$

An important concept to remember is that dB and dBm can be added or subtracted from each other. For example, if a -20 dBm signal is sent to an amplifier with a gain of 30 dB, the output will be

$$-20 \text{ dBm} + 30 \text{ dB} = +10 \text{ dBm}$$

Even though they look like different units, dB and dBm can be added or subtracted because they are really alike. The above equation is equivalent to saying “if you start with a signal which is 20 dB weaker than a milliwatt, and amplify it by 30 dB, the result will be 10 dB stronger than a milliwatt.”

DBm are quite common, although sometimes they are often labelled dB rather than dBm. For example, Fig. A-3 shows the “DB” scale on a very common multimeter. You will note that 0 dB on the scale lines up with 0.775 volts on the ac voltage scale directly above it. The reason is that the original dBm definition applied to 600-ohm circuits, such as those used for analog telephone lines. If we calculate the power with 0.775 volts across 600 ohms, we get

$$P = \frac{V^2}{R} = \frac{0.775^2}{600} = 1 \text{ milliwatt}$$

Hence 0.775 volts in a 600-ohm circuit corresponds to 0 dBm (remember: 0 dBm is neither more nor less than just 1 milliwatt.) Note that this power level would corre-

spond to a different voltage if the resistance was something other than 600 ohms.

dB for Voltage and Current Ratios

Although decibels were originally defined for power calculations, they can also be used for voltage and current. The formulas are similar, except that the constant of 10 in the formula is replaced by a 20; that is, 20 log is used instead of 10 log. (The 10 is required for power calculations to convert bels into decibels; an extra factor of 2 is needed for voltage and current because power is proportional to the square of voltage or current; multiplying a log by 2 is equivalent to squaring.)

For many circuits, power, voltage, and current gains in dB will be different; the three values will only be the same when the circuit resistance is constant. For example, if the input and output load resistances of an amplifier are equal, then its power gain, voltage gain., and current gain (calculated in dB) will be the same. If the resistances are different, then the three dB values will also be different.

For an example, consider an amplifier whose input and output resistances are equal, and whose voltage gain ratio is 2. The voltage gain in dB is therefore

$$\text{dB gain} = 20 \log_{10} 2 = 20 \times 0.3 = 6 \text{ dB}$$

Since the input and output load resistances are equal, if the output voltage is twice the input, so is the output current. Since the same formula is used for current gain, the current gain is also 6 dB. The power, however, is proportional to V^2/R ; R is the same but V is doubled, so the output power is 4 times larger than the input power. The power gain in dB is therefore

$$\text{dB gain} = 10 \log_{10} 4 = 10 \times 0.6 = 6 \text{ dB}$$

Note how the use of a 20 instead of 10 compensates for the squaring of the ratio, so that we still get 6 dB gain.

If, however, the input and output resistances were different, then the power ratio would not be 4, and so the power gain in dB would not equal the voltage gain.

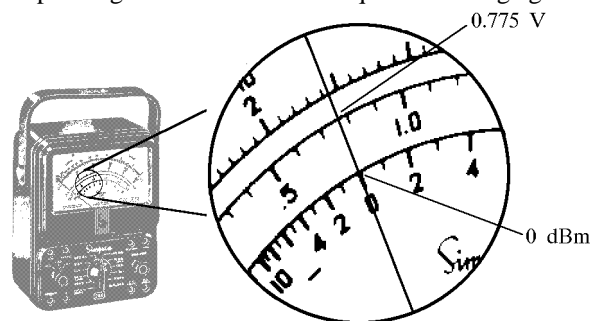


Fig. A-3. The DB scale on a Simpson 260 Multimeter

We can now extend our previous table to include voltage and current DB gains, like this:

Gain ratio	Power gain in dB	Voltage or Current gain in dB
1,000,000	+60	+120
100,000	+50	+100
10,000	+40	+80
1000	+30	+60
100	+20	+40
10	+10	+20
2	+3	+6
1	0	0
.5	-3	-6
.1	-10	-20
.01	-20	-40
.001	-30	-60
.0001	-40	-80
.00001	-50	-100
.000001	-60	-120

Note how the use of 20 log, instead of 10 log, gives double the number of dB for voltage and current, as compared with power.

At this point, it would pay to list a few typical numbers:

- Doubling the power means an increase of +3 dB, but doubling the voltage (or current) is an increase of +6 dB.
- Similarly, cutting the power in half is -3 dB, whereas cutting the voltage (or current) in half is a drop of -6 dB.
- Another voltage ratio that occurs fairly often in practice (even though it is not in the above table) is 0.707, representing a drop to 70.7% of the original voltage. This is important because it represents a loss of 1/2 of the power (since 0.707 squared is 0.5). A voltage ratio of 0.707 translates to -3 dB.

Examples

Before we discuss some common examples of how decibels are used, remember that the dB is used to *compare two numbers*. This can take one of two forms:

- Sometimes both numbers being compared can vary. For example, in measuring the gain of an amplifier, both the input and the output can vary, and the gain is simply the ratio of output over input, converted into decibels.
- Other times, one of the two numbers being compared is a constant — for example, with dBm,

we always compare some power level with a power of 1 milliwatt. The 1-milliwatt power level is a reference, which is defined as 0 dBm. Our examples also fall into those two categories.

Example A-1: Frequency response

The gain of an amplifier or filter often depends on the frequency at which we measure it. A graph of the gain at various frequencies is called the frequency response; an example of the frequency response of a some device is shown in Fig. A-4.

Most such graphs are plotted on a special logarithmic graph paper, where the frequency is not evenly spaced across the graph. In this kind of graph, each decade — that is, a 10-to-1 ratio of frequency — takes up an equal width of paper. As a result, low frequencies are spread out on the sheet, whereas high frequencies are compressed.

In this graph, the gain at low frequencies of perhaps 10 or 20 Hz is 0 dB, which corresponds to a gain of 1. As the frequency rises, the gain also rises, until it reaches a peak of 50 dB at about 4000 Hz; above that frequency, it again rapidly falls, and by 100,000 Hz it is down to about 5 dB. Although higher frequencies are not shown, we can assume that the gain would continue dropping at higher frequencies.

Fig. A-4 would be useful if we wanted to know the actual gain at some particular frequency. For example, the gain at 4000 Hz is 50 dB, which we could convert to a ratio using either

$$50 \text{ dB} = 10 \log \frac{P_{out}}{P_{in}}$$

if we are using power gain, or

$$50 \text{ dB} = 20 \log \frac{V_{out}}{V_{in}}$$

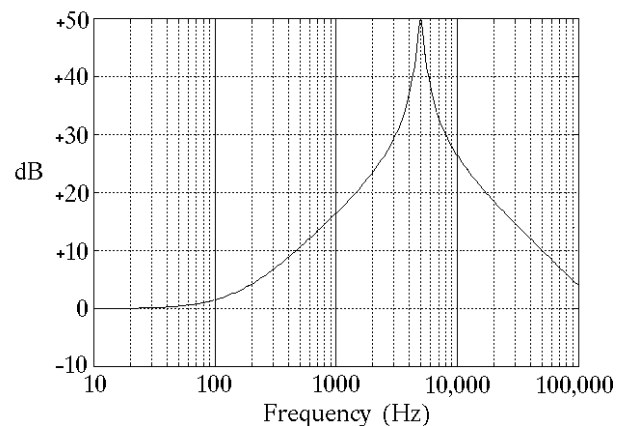


Fig. A-4. Frequency response graph

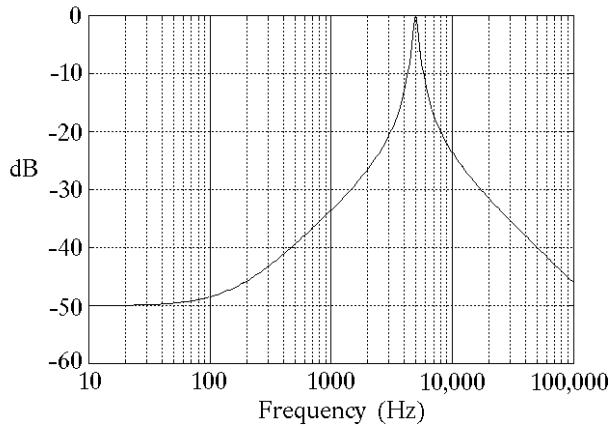


Fig. A-5. Frequency Response but with 0 dB at the top

if we are using voltage gain.

Very often, though, we don't really care about the exact gain, but are instead interested in knowing how the gain changes over some range of frequencies, probably somewhere in the middle. For example, we might like to know how the gain at 3000 Hz compares to that at 4000 Hz. Looking at Fig. A-4, we would proceed as follows:

- (a) The gain at 4000 Hz is 50 dB.
- (b) The gain at 3000 Hz is about 37 dB
- (c) Subtracting 37 from 50, therefore the gain at 3000 Hz is about 13 dB less than the gain at 4000 Hz.

This is OK, but it gets a bit tedious if we want to compare the gains at a number of different frequencies. We can avoid the subtraction if we simply plot that response as in Fig. A-5. Here, we simply define 0 dB as the peak of the curve, and directly read off the gain at 3000 Hz as about -13 dB.

Fig. A-5 no longer tells us the actual gain at any particular frequency; it merely tells us how the gain compares with the gain at, in this case, 4000 Hz. To get an idea of relative gains, we could have put the 0 dB mark at any point on the curve.

Example A-2: Sound Pressure Level

Decibels use logarithms, and the human ear also behaves logarithmically, so it is logical to use decibels to describe sound and how the ear works.

Sound technicians and engineers often need to describe how loud a given sound is. One way is to measure the pressure that the sound exerts on the eardrum. Plumbers often measure pressure in pounds per square inch, but scientists use the metric system and so mea-

sure the sound pressure in Newtons per square centimeter. The weakest sound that the average person can hear is about 0.000002 Newtons per square meter (which is a very slight pressure indeed!)

In a similar way, we could measure the sound pressure level produced by various other sounds around us, but using Newtons per square centimeter results in a wide range of very unwieldy numbers, and so sound experts have simply defined the 0.000002 Newton per square centimeter level as 0 dB SPL (Sound Pressure Level), and measure all other audible sound levels in dB above that weakest sound. The 0 dB SPL value is called the *threshold of hearing*, and the following table shows the approximate dB SPL for some other common sounds:

dB SPL	Sound Source
180	Rocket launch (very close!)
140	Jet engine at takeoff (25 or 50 feet away)
120	Rock concert, nearby thunderclap
100	Nearby firecracker, motorcycle
80	Nearby freight train, factory noise
60	Nearby conversation, office noise
40	Quiet office or library
20	Quiet outside area, rustling leaves
10	Quiet breathing
0	Threshold of hearing

Clearly, sounds near 0 dB SPL are barely audible, while sounds above 100 dB SPL are quite loud. In fact, sounds above 80 dB SPL can cause hearing damage, especially for longer periods; somewhere around 120 or 130 dB SPL is considered the threshold of pain, and 180 dB SPL can cause the eardrum to rupture.

Example A-3: dBV and dBu

Just as dBm compares power levels to a standard power of 1 milliwatt, so dBV compares a voltage level to a 1-volt rms signal. Thus 0 dBV is 1 volt, +20 dBV is 10 volts, and -10 dBV is 0.316 volts (10 dB less than 1 volt).

dBu also uses a voltage as reference, but its 0 dBu level is 0.775 volts. Why such an unusual number? Recall that 0.775 volts rms in 600 ohms represents a power of 1 milliwatt, which is 0 dBm. Thus, in a 600-ohm load, the reading in dBu and dBm will be the same! 600 ohms is used because that was the line impedance in early Bell telephone systems.

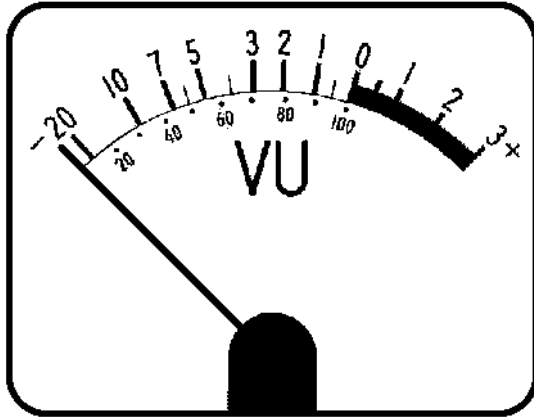


Fig. A-6. The VU meter

Example A-4: Volume Units (VU)

Broadcast and recording studio technicians want to record sound as loud as their equipment can take without significant distortion or clipping. Fig. A-6 shows a common meter that they use to set their sound level — it is called a *VU meter*.

0 VU on the scale is defined as the maximum desirable sound level (whatever that may be). The scale to the left goes down into negative VU, while the scale to the right (which is usually painted in red) typically goes up to about +3 VU. Sound technicians generally adjust the volume so that the VU meter indicates a level as close to 0 VU, but not past it into the red area to its right.

Although the meter is clearly labelled in VU, it actually measures the audio signal voltage, and is calibrated logarithmically in decibels. Remember that a 6 dB voltage difference is a 2-to-1 ratio. If you carefully examine the scale in Fig. A-6, you will see that -6 VU is about halfway toward 0 VU. Likewise, -12 VU is halfway to -6 VU, -18 VU is about halfway to -12 VU, and so on. Thus a difference of 6 VU represents a 2-to-1 voltage ratio, same as decibels do.

But, aside from the fact that 0 VU is the maximum desirable audio level, how much signal does 0 VU really represent? That actually depends on the equipment that the recording or broadcast studio uses, but typical 0 VU levels used are either +4 dBu (a voltage of 1.23 volts RMS which, in a 600-ohm circuit, is the same as +4 dBm), or -10 dBV (which is 0.316 volts). The exact voltage level doesn't really matter, as long as all the equipment in a given studio uses the same 0 VU reference voltage.

Example A-5: dBi and dBd

As we see in the chapter on wireless, antennas are used to transmit and receive radio, TV, and other wireless signals. One way to measure the quality of an antenna (especially a complicated one) is to compare it with some other, more basic kind of antenna. This is where the dBi and dBd comes in.

The dBi rating compares an antenna to a theoretical antenna called an isotropic antenna. For example, an antenna rated as +3 dBi sends, or receives, a signal that is 3 dB stronger than an isotropic antenna would have.

The dBd, on the other hand, compares an antenna to a dipole. For example, an antenna rated as +3 dBd sends, or receives, a signal that is 3 dB stronger than that of a dipole.

It turns out that the dipole is better than an isotropic antenna — we know that a dipole is rated as 2.14 dBi, meaning that it is 2.14 dB better than an isotropic antenna. That means that an antenna rated as 3 dBd (3 dB better than the dipole) is actually 3 + 2.14, or 5.14 dB better than the isotropic, so it would be rated as 5.14 dBi.